Ph.D. Qualifying Exam
CS 500 Algorithms Design and Analysis
Winter 2009/2010

You must answer in *English*. Write clearly and readably!

A *directed* graph $G = (V, E)$ is called *acyclic* if it does not contain a directed cycle.

A *topological ordering* of a directed graph $G = (V, E)$ is an ordering $v_1, v_2, \ldots, v_n$ of the vertices V such that for every directed edge $(v_i, v_j) \in E$ we have $i < j$.

Problem 1. (50 points) Prove that a directed graph $G = (V, E)$ is acyclic if and only if there exists a topological ordering of G.

Problem 2. (50 points) Consider the following problem:

> *Given a directed graph $G = (V, E)$, decide whether it is acyclic.*

Give a short argument that uses the result from problem 1 to show that this problem is in NP.