CS520 Theory of Programming Languages

(Theory Area)

Phd. Qualifying Exam (January 12, 2019) - **OPEN BOOK**

Problem 1 (40 points)

Consider the syntax of the language E for the "palm calculator".

E	$\rightarrow$	n	number
	$\mid$	x	variable
	$\mid$	$E + E$	addition
	$\mid$	$E - E$	subtraction
	$\mid$	$(\; E \;)$	parenthesis
	$\mid$	let $x = E$ in E	binding

We will implement this language E with <*S,E,C*>-machine. The <*S,E,C*>-machine is an abstract machine. S is a stack of values (ordered sequence). E is an environment (function : Variable $\rightarrow$ Value). C is a command sequence defined as following:

C	$\rightarrow$	add$\cdot C$	/* add top two values and push the result back */
	$\mid$	sub$\cdot C$	
	$\mid$	bind$(x)\cdot C$	/* bind x in E with the value of stack-top, and pop */
	$\mid$	unbind$(x)\cdot C$	/* delete the most-recent binding x in E */
	$\mid$	push$(x)\cdot C$	/* push the value of the variable x */
	$\mid$	push$(n)\cdot C$	/* push the constatnt n */
	$\mid$	ε	; empty command

(a) Define the denotational semantics of the language E

(b) Define the transition semantics (abstract machine semantics) of the command C.
If you need a new command, define its syntax and transition semantics freely.
You may use your own function if it is *"well-defined"*.

(c) Define the compilation rules $\triangleright$ from the program E to the command C sequence with inference rules

Problem 2 (40 points)

Let **P** be the vertical domain of the natural numbers, $\{0, 1, 2, \ldots, \infty\}$ where

$$0 \sqsubseteq 1 \sqsubseteq 2 \sqsubseteq 3 \quad \sqsubseteq \ldots \quad \sqsubseteq \infty,$$

and **P′** the two-element domain $\{\bot, \top\}$ where $\bot \sqsubseteq \top$.

Then the monotone function $\boldsymbol{f\,x} \equiv \textbf{if } \boldsymbol{x} = \infty \textbf{ then } \top \textbf{ else } \bot$ is not continuous.

(a) Why is the function f "not continuous"? Explain.

(b) Let a function f_k be $f_k\ x \equiv$ **if** $x \leq k$ **then** $\bot$ **else** $\top$. Then, is the function f_k continuous? Explain why or why not?

(c) Does a sequence $f_0\ f_1\ f_2\ldots$ constitute a chain? Why or why not? What is the least element among f_k where $k = 0, 1, \ldots, \infty$?
($f \sqsubseteq g$ iff $\forall x \in \mathbf{P}.\ f\ x \sqsubseteq g\ x$)

Problem 3 (20 points)

The set T is defined to consist only of t that are inductively defined as follows:

$$t \rightarrow \bullet \mid /\ t\ ,\ t\ /$$

Every $t \in T$ enjoys several invariant conditions on the numbers of commas and slashes in t. Prove by induction that the number of slashes in t is (2 * the number of commas) for any element of T. First, state your theorem formally(with some mathematical notation), then prove your theorem by induction.