CS520 Theory of Programming Languages

(Theory Area)

Phd. Qualifying Exam (July 7, 2017) - **OPEN BOOK**

Problem 1 (40 points)

Consider the syntax of the language E for the "palm calculator".

E	$\rightarrow$	n	number
	|	x	variable
	|	$E + E$	addition
	|	$E - E$	subtraction
	|	$(\; E \;)$	parenthesis
	|	let x = E in E	binding

We will implement this language E with <*S,E,C*>-machine. The <*S,E,C*>-machine is an abstract machine. S is a stack of values (ordered sequence). E is an environment (function : Variable $\rightarrow$ Value). C is a command sequence defined as following:

C	$\rightarrow$	add·C	/* add top two values and push the result back */
	|	sub·C	
	|	bind(x)·C	/* bind x in E with the value of stack-top, and pop */
	|	unbind(x)·C	/* delete the most-recent binding x in E */
	|	push(x)·C	/* push the value of the variable x */
	|	push(n)·C	/* push the constatnt n */
	|	ε	; empty command

(a) Define the denotational semantics of the language E

(b) Define the transition semantics (abstract machine semantics) of the command C.
If you need a new command, define its syntax and transition semantics freely.
You may use your own function if it is *"well-defined"*.

(c) Define the compilation rules $\triangleright$ from the program E to the command C sequence with inference rules

Problem 2 (40 points)

Let **P** be the vertical domain of the natural numbers, $\{0, 1, 2, \ldots, \infty\}$ where

$$0 \sqsubseteq 1 \sqsubseteq 2 \sqsubseteq 3 \sqsubseteq \ldots \sqsubseteq \infty,$$

and **P′** the two-element domain $\{\bot, \top\}$ where $\bot \sqsubseteq \top$.

Then the monotone function $\boldsymbol{f\,x} \equiv$ **if** $\boldsymbol{x} = \infty$ **then** $\top$ **else** $\bot$ is not continuous.

(a) Why is the function $\boldsymbol{f}$ "<u>not continuous</u>"? Explain.

(b) Let a function $\boldsymbol{f_k}$ be $\boldsymbol{f_k\ x} \equiv$ **if** $\boldsymbol{x \geq k}$ **then** $\bot$ **else** $\top$. Then, is the function $\boldsymbol{f_k}$ continuous? Explain why or why not?

(c) Does a sequence $\boldsymbol{f_0\ \ f_1\ \ f_2 \ldots}$ constitute a chain? Why or why not? What is the least element among $\boldsymbol{f_k}$ where $\boldsymbol{k} = 0, 1, \ldots, \infty$?
($\boldsymbol{f} \sqsubseteq \boldsymbol{g}$ iff $\forall \boldsymbol{x} \in \mathbf{P}.\ \boldsymbol{f\ x} \sqsubseteq \boldsymbol{g\ x}$)

Problem 3 (20 points)

Set $D \in \mathbb{N} \times \mathbb{N}$ consists only of the elements (n, m) that are inductively defined as follows:

$$\frac{}{(n,\ n) \in D} \qquad\qquad \frac{(n,\ m) \in D}{(n,\ n \times m) \in D}$$

Prove that

For any $(n, m) \in D$, there exists $i \in \mathbb{N}$ such that $m = n^{i}$.