1. What is the worst-case running time of the algorithm for performing a 3-dimensional range query on a kd-tree storing n points in 3D? Justify your answer.

2. (a) Show that the set of edges of a Delaunay triangulation of a set P of points in the plane contains the Euclidean minimum spanning tree (EMST) of P.

(b) Use this result to give an efficient algorithm to compute an EMST of P. Analyze the running time of your algorithm.

3. What is the lower bound for computing the convex hull of a set of n points in the plane? Justify.

4. Let \( p := (p_x, p_y) \) be a point in the plane. The dual of \( p \), denoted \( p^* \), is the line defined as \( p^* := (y = p_x x - p_y) \). The dual of a line \( l : y = mx + b \) is the point \( p \) such that \( p^* = l \). In other words, \( l^* := (m, -b) \).

   (a) Prove that the above duality transform is incidence and order preserving.

   (b) What is the dual of a line segment whose endpoints are \( p \) and \( y \)?

   (c) What is the dual of the collection of points inside a given triangle with vertices \( p, q, \) and \( r \)?