CS520 Theory of Programming Languages

(Theory Area)

Phd. Qualifying Exam (January 13, 2017)

Problem 1 (40 points)

Consider the syntax of the language E for the "palm calculator".

E	$\rightarrow$	n	number
	$\vert$	x	variable
	$\vert$	$E + E$	addition
	$\vert$	$E - E$	subtraction
	$\vert$	$(\; E \;)$	parenthesis
	$\vert$	let $x = E$ in E	binding

We will implement this language E with $<S,E,C>$-machine. The $<S,E,C>$-machine is an abstract machine. S is a stack of values (ordered sequence). E is an environment (function : Variable $\rightarrow$ Value). C is a command sequence defined as following:

C	$\rightarrow$	add$\cdot C$
	$\vert$	sub$\cdot C$
	$\vert$	bind$(x)\cdot C$
	$\vert$	unbind$(x)\cdot C$
	$\vert$	push$(x)\cdot C$
	$\vert$	push$(n)\cdot C$
	$\vert$	e ; empty command

(a) Define the denotational semantics of the language E

(b) Define the transition semantics (abstract machine semantics) of the command C.

(c) Define the compilation rules $\triangleright$ from the program E to the command C sequence with inference rules

Problem 2 (40 points)

Let **P** be the vertical domain of the natural numbers, $\{0, 1, 2, \ldots, \infty\}$ where

$$0 \sqsubseteq 1 \sqsubseteq 2 \sqsubseteq 3 \sqsubseteq \ldots \sqsubseteq \infty,$$

and **P′** the two-element domain $\{\bot, \top\}$ where $\bot \sqsubseteq \top$.

Then the monotone function $\boldsymbol{f\ x} \equiv$ **if** $\boldsymbol{x} = \infty$ **then** $\top$ **else** $\bot$ is not continuous.

(a) Why is the function $\boldsymbol{f}$ "<u>not continuous</u>"? Explain.

(b) Let a function $\boldsymbol{f_k}$ be $\boldsymbol{f_k\ x} \equiv$ **if** $\boldsymbol{x \leq k}$ **then** $\bot$ **else** $\top$. Then, is the function $\boldsymbol{f_k}$

continuous? Explain why or why not?

(c) Does a sequence f_0 f_1 f_2... constitute a chain? Why or why not? What is the least element among f_k where k = 0, 1, … , ∞?

($f \sqsubseteq g$ iff $\forall x \in \mathbf{P}.\ f\ x \sqsubseteq g\ x$)

Problem 3 (20 points)

Set $D \subseteq \mathbb{N} \times \mathbb{N}$ consists olny of the elements (n, m) that are inductively defined as follows:

$$\frac{}{(n,\ n) \in D} \qquad\qquad \frac{(n,\ m) \in D}{(n,\ n \times m) \in D}$$

Prove that

For any $(n, m) \in D$, there exists $i \in \mathbb{N}$ such that $m = n^{i}$.