

CS504 PhD qualifying exam

1. Prove or disprove :

If  $X$  is a convex set and  $Y$  is a convex set, then  $X \cap Y$  is a convex set.

2. What is the lower bound for computing the convex hull of a set of  $n$  points in the plane? Justify.

3. Let  $p := (p_x, p_y)$  be a point in the plane. The dual of  $p$ , denoted  $p^*$ , is the line defined as  $p^* := (y = p_x x - p_y)$ . The dual of a line  $l : y = mx + b$  is the point  $p$  such that  $p^* = l$ . In other words,  $l^* := (m, -b)$ .

(a) Prove that the above duality transform is incidence and order preserving.

(b) What is the dual of a line segment whose endpoints are  $p$  and  $y$ ?

(c) What is the dual of the collection of points inside a given triangle with vertices  $p$ ,  $q$ , and  $r$ ?