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CS500 July 1st, 2016 PhD Qualifying Exam

Please write your name and student ID here ______________________________

as well as on each additional sheet of paper you use!

100 points = 100%

Problem 1 (5+5+5+5 points):

a) Which of the following* are *sound* measures of algorithmic cost?

- ☐ software licence fee/purchase cost
- ☐ programmers' salaries
- ☐ runtime (#CPU seconds)
- ☐ runtime (#steps)
- ☐ asymptotic (big-O) #steps
- ☐ asymptotic memory consumption (#bits)
- ☐ asymptotic communication volume (#bits)
- ☐ asymptotic #processors · #parallel steps
- ☐ energy consumption (#kWh)
- ☐ #bugs

b) Which of the following* are *sound* notions of an algorithm's performance?

- ☐ worst-case
- ☐ best-case
- ☐ typical case
- ☐ average-case†
- ☐ in practice
- ☐ on a benchmark
- ☐ amortized
- ☐ expected (for randomized algorithms)
- ☐ accuracy (for approximation algorithms)
- ☐ competitive (for online algorithms)

c) Explain the differences between (i) a program, (ii) an algorithm, and (iii) a heuristic.

d) Explain the difference between (i) algorithmic cost and (ii) computational complexity.

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* Check your multiple choice answers on this paper: $\frac{1}{2}$ point for each correct, 0 for each incorrect
† with respect to a certain probability distribution on the space of inputs...

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Problem 2 (5+ 5+ 5+ 5 points):

a) Specify (!) and describe three *significantly* different algorithms for sorting n given keys, together with their asymptotic computational worst-case costs. (No proofs required.)

b) Asymptotically analyze the (#arithmetic operations used by the) high-school method (a.k.a. long multiplication) for calculating, given the coefficients $a_0,\ldots,a_n$ and $b_0,\ldots,b_n$ of univariate polynomials $A(x) = a_0+a_1\cdot x+a_2\cdot x^2+\ldots+a_n\cdot x^n$ and $B(x) = b_0+b_1\cdot x+b_2\cdot x^2+\ldots+b_n\cdot x^n$, determine the coefficients $c_0,\ldots,c_{2n}$ of their product polynomial $C(x) := A(x)\cdot B(x)$.

c) Verify the correctness of the following formula. Describe a recursive algorithm based on it for the problem from **b)**.

$$(A_0(x) + A_1(x)\cdot x^n)\cdot(B_0(x) + B_1(x)\cdot x^n) \;=\; C_0(x) + C_1(x)\cdot x^n + C_2(x)\cdot x^{2n},$$

where $C_0(x) := A_0(x)\cdot B_0(x)$, $C_2(x) := A_1(x)\cdot B_1(x)$, and

$$C_1(x) := (A_0(x) + A_1(x))\cdot(B_0(x) + B_1(x)) - C_0(x) - C_2(x)$$

d) Analyze the asymptotic runtime (=#arithmetic operations) of your algorithm from **c)**.

Problem 3 (10× 2 points): Match[‡] the algorithms/problems on the left to their least (known) among the classes of asymptotic worst-case runtime/time complexity to the right:

- Binary search among n sorted elements •
- Comparison-based sorting •
- Connectedness of a given graph •
- **Vertex Cover** (Problem 5) •
- **Edge Cover**: Given a graph $G=(V,E)$ and $k\in\mathbb{N}$, do there exist edges $e_1,\ldots,e_k\in E$ s.t. every vertex $v\in V$ belongs to some $e\in\{e_1,\ldots,e_k\}$? •
- **Minimum Spanning Tree** of a given connected graph with n vertices and $O(n)$ edges •
- Multiplication of two $n\times n$ matrices of entries `0`,`1` •
- Syntax test (parsing) w.r.t. a regular grammar •
- Syntax test w.r.t. a context-free grammar •
- Searching a given string of length n for the occurrence of a given substring of length $O(n)$ •

▪ $O(\log^2 n)$
▪ $O(\sqrt{n})$
▪ $O(n)$
▪ $O(n\cdot\log^2 n)$
▪ $O(n^2\cdot\log^2 n)$
▪ $O(n^3\cdot\log^2 n)$
▪ $\mathcal{P}$
▪ $\mathcal{NP}$

[‡] Draw your answers on this paper: 2 points for each correct line, 0 for each missing/incorrect one

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Problem 4 (5+5+5+5 points):

a) What is the asymptotic (i) *worst-case* and (ii) *amortized* cost of incrementing a binary counter, when each bit-flip counts as one step ? (No proof is required here…)

b) Prove your second claim from **a)**.

c) Determine the *average* cost of the following fun algorithm, asymptotically as $n \to \infty$:

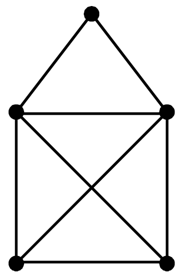
Given a tuple $(b_1,\ldots,b_n)$ of n bits, search the (index j of the) first non-zero bit b_j; in case all b_j are zero, count to 2^n-1 and stop.

d) What is the (i) *worst* and (ii) *expected* cost of the following randomized 'algorithm':

Flip a coin. If it comes out ***heads***, stop; otherwise repeat.

Hint: It holds $\sum_n n \cdot p^n = p/(1-p)^2$ for all $|p|<1$.

Problem 5 (5+5+5+5 points): Recall that *Vertex Cover* is the following optimization problem: Given an undirected graph $G=(V,E)$, find the least number $k=k(G)$ of vertices $v_1,\ldots,v_k \in V$ such that every edge $e \in E$ is incident to (i.e. has among its two end points) at least one vertex from the set $C=\{v_1,\ldots,v_k\}$. The corresponding decision problem asks whether, given G and ℓ, it holds $k(G) \leq \ell$.



a) Determine $k(G)$ and an optimal *Vertex Cover* for the following graph G:

b) Consider the following greedy algorithm, initialized with $C=\{\}=F$:

WHILE there exists an edge $e=\{a,b\} \in E$,
add e to F and both its end points a,b to C
and remove from E all edges incident to a or b.

Prove that the resulting set C constitutes a vertex cover of size $2 \cdot |F| \leq 2 \cdot k(G)$, i.e. a 2-approximation.

c) Prove that the analysis in **b)** is optimal by constructing (a family of) graphs G where the above algorithm produces a vertex cover of size $2 \cdot k(G)$.

d) Will the following variant of **b)** also yield a vertex cover and which approximation ratio?

For each edge $e=\{a,b\} \in E$ add only *one* (arbitrary) of its end points to C
and remove from E all edges incident to that vertex.

Justify your answers!