[박사과정 자격 시험 문제: CS572 지능형 로보틱스]

1. (10 pts)

   Let's think a domain consisting of three binary variables *Toothache*, *Cavity*, and *Catch*, and assume the full joint probability distribution is a table as follows.

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

   Please, compute the following probabilities.

   - P(*cavity*)=?

   - P(*cavity* | *toothache*,)=?

   - P(*cavity* | *toothache*, *¬catch*)=?

   - P(*toothache* | *cavity*, *¬catch*)=?

2. (30 pts)

   For each of the following statements, determine whether it is TRUE or FALSE. Make sure you give a sufficient but brief justification of your answers. Notice that each statement is independent of the others, so you cannot use the assumptions of one in the others.

   (1) If random vectors x and y are joint Gaussian and *E[x|y]=E[x]*, then x and y are statistically independent.

   (2) Given two random variables *x* and *y*, if *E[x|y]=E[x]*, then for all functions *f(.)*, *E[xf(y)]]=E[x]E[f(y)]* (whenever theses expectation are defined).

   (3) Given two random variables *x* and *y*, if *E[xf(y)]=E[x]E[f(y)]* for all functions *f(.)*, then *E[x|y]=E[x].*

3. (30pts)

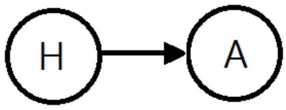

Figure 1.

Figure 1 shows a simple Bayesian network. H and A indicate hidden and observed binary variables, respectively, whose value is either 0 or 1. Throughout the simulations, we observed 30 times when A is 0, and 70 times when A is 1. We would like to use the EM algorithm to estimate probability distributions.

Set P(H=1), P(A=1|H=0), and P(A=1|H=1) to $\theta_1(t)$, $\theta_2(t)$, and $\theta_3(t)$ respectively as parameters at t-th iteration. We assume the initial parameter values as follows.

$\theta_1(0) = 0.3$, $\theta_2(0) = 0.2$, and $\theta_3(0) = 0.6$

Please, computer the three parameter values until t=2, that is to say, $\theta_1(1)$, $\theta_2(1)$, $\theta_3(1)$, $\theta_1(2)$, $\theta_2(2)$, and $\theta_3(2)$, while the EM algorithm operates.

4. (30pts)

Consider a linear quadratic regulator problem that minimizes

$J = \frac{1}{2}x(t_f)^T Q_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} x(t)^T Q(t)x(t) + u(t)^T R(t)u(t)dt$

subject to $\dot{x}(t) = A(t)x(t) + b(t)u(t)$.

The optimal control should be a feedback term.

$u^*(t) = -K(t)x(t)$.

Using the Hamilton-Jacobi-Bellamn (HJB) equation, show that a possible optimal value function is of the form

$$V^*(x(t), t) = \frac{1}{2}x(t)^T K(t)x(t)$$

In the process demonstrate that the following condition must be satisfied.

$$-K = Q(t) + K(t)A(t) + A(t)^T K(t) - K(t)B(t)R(t)^{-1}B(t)^T K(t)$$

with a boundary condition: $K(t_f) = Q_f$.