Qualifying Exam for Algorithm. Jan., 2016.

1. (20 pts) Give an efficient algorithm for finding all-pairs shortest paths of a directed graph with real-numbered edge weights. Analyze the running time of your algorithm and show correctness. (A more efficient algorithm will earn more points.)

2. (30 pts) Given a directed graph G, we say that a cycle C in G is a Hamiltonian cycle if it visits each vertex exactly once. The Hamiltonian cycle problem is then the following :

   Given a directed graph G, does it contain a Hamiltonian cycle?

   (1) Is the Hamiltonian cycle problem NP? Justify.
   (2) Is the Hamiltonian cycle problem NP-hard? Justify.
   (3) Is the Hamiltonian cycle problem NP-complete? Justify.

3. (10 pts) The running time of QUICKSORT depends on both the data being sorted and the partition rule used to select the pivot. Suppose we always pick the pivot element to be the key of the median element of the first three keys of the subarray. On a sorted array, determine whether QUICKSORT now takes $\Theta(n)$, $\Theta(n \log n)$, or $\Theta(n^2)$. Justify your answer.

4. (10 pts) Solve the following recurrence by giving tight Θ-notation bounds. Justify your answer.

$$T(n) = 7T\left(\frac{n}{2}\right) + n^2$$

5. (30 pts) Give True ot False for each of the following statements. Justify your answers. Incorrect justification will earn 0 pts even with correct answer.

(1) A graph algorithm with $O(E \log V)$ running time is asymptotically better than an algorithm with $O(E \log E)$ running time for a connected, undirected graph $G(V, E)$.
(2) For every two positive functions f and g, if $g(n) = O(n)$, then $f(g(n)) = O(f(n))$.
(3) Every 2-approximation algorithm for finding a minimum vertex cover is also a 2-approximation algorithm for finding a maximum independent set.