박사과정 자격 시험 문제: CS572 지능형 로보틱스

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1. (10 pts)

   Let's think a domain consisting of three binary variables *Toothache*, *Cavity*, and *Catch*, and assume the full joint probability distribution is a table as follows.

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

   Please, compute the following probabilities.

   - P(*cavity*)=?
   - P(*cavity* | *toothache*,)=?
   - P(*cavity* | *toothache*, *¬catch*)=?
   - P(*toothache* | *cavity*, *¬catch*)=?

2. (30 pts)

   For each of the following statements, determine whether it is TRUE or FALSE. Make sure you give a sufficient but brief justification of your answers. Notice that each statement is independent of the others, so you can not use the assumptions of one in the others.

   (1) If random vectors x and y are joint Gaussian and E[x|y]=E[x], then x and y are statistically independent.

   (2) Given two random variables x and y, if E[x|y]=E[x], then for all functions f(.), E[xf(y)]]=E[x]E[f(y)] (whenever theses expectation are defined).

   (3) Given two random variables x and y, if E[xf(y)]=E[x]E[f(y)] for all functions f(.), then E[x|y]=E[x].

3. (30pts)

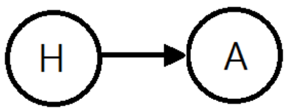

Figure 1.

Figure 1 shows a simple Bayesian network. H and A indicate hidden and observed binary variables, respectively, whose value is either 0 or 1. Throughout the simulations, we observed 80 times when A is 0, and 50 times when A is 1. We would like to use the EM algorithm to estimate probability distributions.

Set P(H=1), P(A=1|H=0), and P(A=1|H=1) to $\theta_1(t)$, $\theta_2(t)$, and $\theta_3(t)$ respectively as parameters at *t*-th iteration. We assume the initial parameter values as follows.

$\theta_1(0) = 0.3$, $\theta_2(0) = 0.2$, and $\theta_3(0) = 0.6$

Please, computer the three parameter values until *t*=2, that is to say, $\theta_1(1)$, $\theta_2(1)$, $\theta_3(1)$, $\theta_1(2)$, $\theta_2(2)$, and $\theta_3(2)$, while the EM algorithm operates.

4. (30pts)

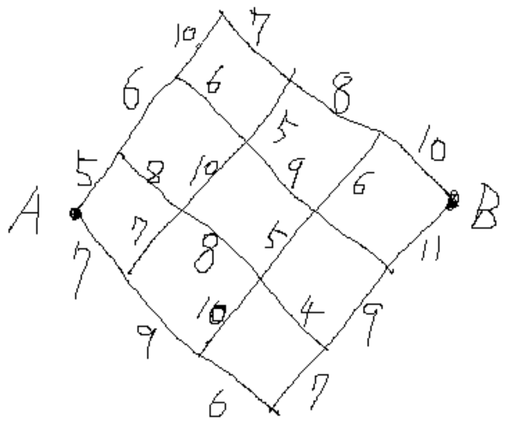

Figure 2.

This problem aims to solve a multistage optimization problem shown in Figure 2. We would to find an optimal path from A to B. Each node indicates a state. Left-to-right move is only possible by choosing either upward or downward. A digit near each edge indicates a cost value caused while travelling the edge. Our goal is to find a path from A to B which causes a minimal total cost. A path means a sum of travelled edges. In the answer sheet, draw the figure 2 and indicates the optimal path. Also, explain how to find the optimal path simply.