

Ph.D. Qualifying Exam — Algorithms (CS500)

Summer 2015

You must answer in *English*. Write clearly and readably!

Problem 1. You are given a linear program in canonical form:

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{such that} & Ax \leq b \\ & x \geq 0 \end{array}$$

- (a) (20 points) What is the dual program for this linear program?
- (b) (20 points) State the weak duality theorem for the two linear programs.
- (c) (20 points) Prove the weak duality theorem for the two linear programs.

Problem 2. (20 points) Let $G = (V, E)$ be a graph. A *dominating set* in G is a subset S of the vertices such that every vertex in G is either in S or adjacent to a vertex in S . The DOMINATINGSET problem asks, given a graph G and an integer k as input, whether G contains a dominating set of size k .

The DOMINATINGSET problem is known to be NP-hard (you do not need to prove this!). Show that the DOMINATINGSET problem is also NP-complete.

Problem 3. (20 points) You are given a problem X . You wish to prove that X is NP-hard. Which of the following polynomial-time reductions can you use to prove this?

- (A) $X \leq 3\text{-SAT}$
- (B) $2\text{-SAT} \leq X$
- (C) $\text{ShortestPath} \leq X$
- (D) $X \leq \text{VertexCover}$
- (E) $\text{VertexCover} \leq X$
- (F) $X \leq \text{ShortestPath}$