박사과정 자격 시험 문제: CS572 지능형 로보틱스

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1. (15 pts)

Let's think a domain consisting of three binary variables *Toothache*, *Cavity*, and *Catch*, and assume the full joint probability distribution is a table as follows.

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Please, compute the following probabilities.

- P(*cavity*)=?

- P(*cavity* | *toothache*,)=?

- P(*cavity* | *toothache*, *¬catch*)=?

- P(*toothache* | *cavity*, *¬catch*)=?

2. (25 pts)

Let we have three components required to describe a HMM: $P(X_0)$ a prior probability distribution over the states at time 0, $P(X_t|X_{t-1})$ transition model, and $P(E_t|X_t)$ observation model, where X_t, E_t, and e_t describe hidden states, observed variables, and actual observations at time t respectively. Please, derive a recursive formula to estimate the belief state $P(X_t|E_{1:t})$ at each time t based on a previous belief state and a new observation e_t. Make sure to indicate what property or probability rule is applied to at each procedural step to gain the full points.

3. (35pts)

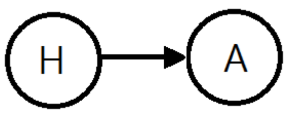

Figure 1.

Figure 1 shows a simple Bayesian network. H and A indicate hidden and observed binary variables, respectively, whose value is either 0 or 1. Throughout the simulations, we observed 100 times when A is 0, and 50 times when A is 1. We would like to use the EM algorithm to estimate probability distributions.

Set P(H=1), P(A=1|H=0), and P(A=1|H=1) to $\theta_1(t)$, $\theta_2(t)$, and $\theta_3(t)$ respectively as parameters at t-th iteration. We assume the initial parameter values as follows.

$\theta_1(0) = 0.4$, $\theta_2(0) = 0.25$, and $\theta_3(0) = 0.54$

Please, computer the three parameter values until t=2, that is to say, $\theta_1(1)$, $\theta_2(1)$, $\theta_3(1)$, $\theta_1(2)$, $\theta_2(2)$, and $\theta_3(2)$, while the EM algorithm operates.

4. (25pts)

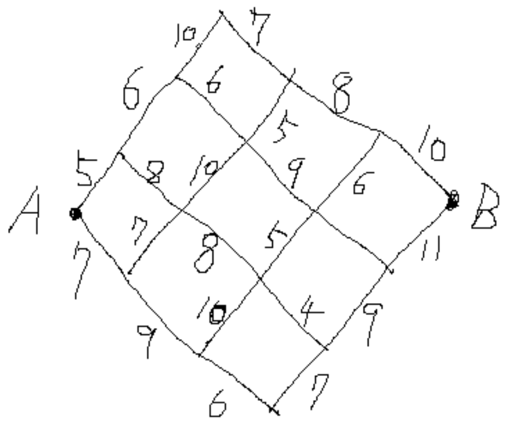

Figure 2.

This problem aims to solve a multistage optimization problem shown in Figure 2. We would to find an optimal path from A to B. Each node indicates a state. Left-to-right move is only possible by choosing either upward or downward. A digit near each edge indicates a cost value caused while travelling the edge. Our goal is to find a path from A to B which causes a minimal total cost. A path means a sum of travelled edges. In the answer sheet, draw the figure 2 and indicates the optimal path. Also, explain how to find the optimal path simply.