1. (10 pts) If P = NP, is NP = co-NP? Justify your answer.

2. (5 pts x 4 = 20 pts) Given 2 decision problems L₁ and L₂ in NP and L₂ \leq_p L₁ (i.e., L₂ is polynomial-time reducible to L₁), for each of the following statements, give one of T(true), F(false), or O(open question), and briefly justify your answer. (NPC denotes the set of NP-complete problems.)
   (1) If \( L₁ \in P \), then \( L₂ \in P \).
   (2) If \( L₂ \in P \), then \( L₁ \in P \).
   (3) If \( L₁ \in NPC \), then \( L₂ \in NPC \).
   (4) If \( L₂ \in NPC \), then \( L₁ \in NPC \).

3. (20 pts) Give a dynamic programming algorithm for finding all-pairs shortest paths of a directed weighted graph and analyze your algorithm. (A more efficient algorithm will earn more points.)

4. Consider the following divide-and-conquer algorithm to multiply two n-bit numbers X and Y. We divide X into A and B and Y into C and D, where A, B, C, D are n/2-bit numbers.

\[
X = 2^{n/2}A + B \\
Y = 2^{n/2}C + D \\
XY = 2^n AC + 2^{n/2} BC + 2^{n/2} AD + BD
\]

(a) (10 pts) Give a recurrence for the running time of the algorithm and give a tight asymptotic bound.

(b) (10 pts) Observe that we can also obtain XY as follows: \( XY = \)

\[
(2^n - 2^{n/2})AC + 2^{n/2}(A + B)(C + D) + (1 - 2^{n/2})BD
\]

Give a recurrence for the running time of this algorithm and give a tight asymptotic bound.
4. (30 pts) Given a directed graph $G$, we say that a cycle $C$ in $G$ is a Hamiltonian cycle if it visits each vertex exactly once. The Hamiltonian cycle problem is then the following:

Given a directed graph $G$, does it contain a Hamiltonian cycle?

(1) Is the Hamiltonian cycle problem NP? Justify.
(2) Is the Hamiltonian cycle problem NP-hard? Justify.
(3) Is the Hamiltonian cycle problem NP-complete? Justify.