KAIST

2014

CS580 Computer Graphics

PhD Qualification Exam

Name _____

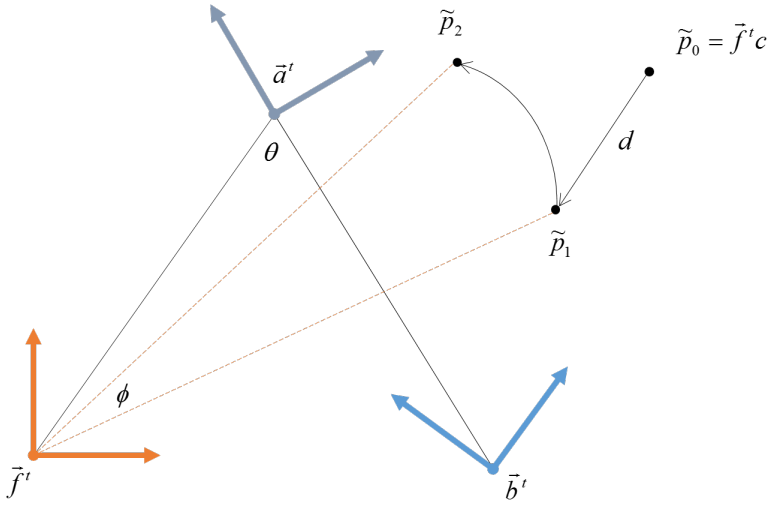
Student ID # _____

Department _____

Questions	Scores
1	
2	
3	
Total	

N.B. Before answering the questions, please look at the entire question set. You may want to write your answers as clear as possible. You can answer in English or Korean. No calculators. You may leave answers in the form of numerical expressions, e.g., such as "cos(2.5/3.7)(1+exp(-4/500))". This includes equations for vectors that include matrix inverses, as long as the elements in the matrix are expressed numerically.

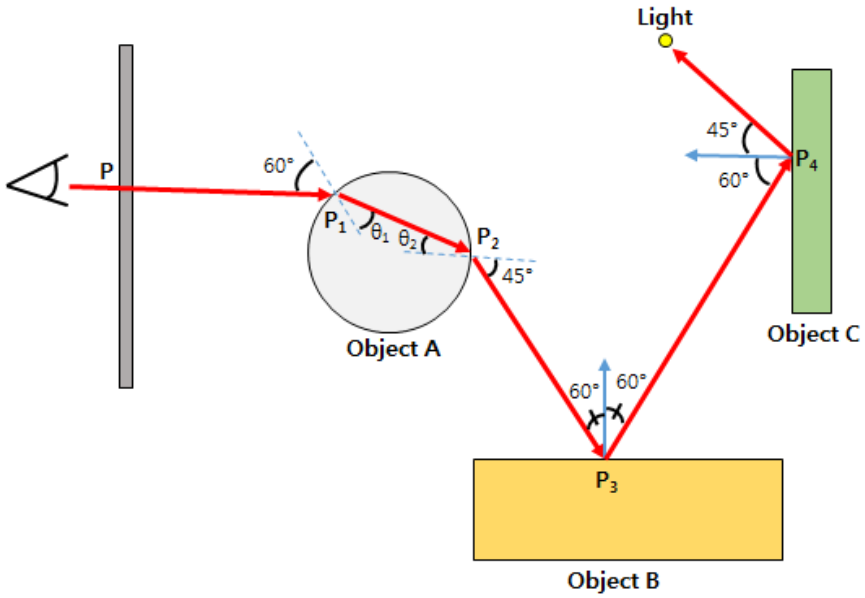
1. (20pts) Given the following three orthonormal frames



- The frame $\vec{b}^t$ is given by rotating the frame $\vec{f}^t$ by θ with respect to $\vec{a}^t$.
- The point $\tilde{p}_1$ is given by translating the point $\tilde{p}_0$ by d in negative x direction with respect to the frame $\vec{b}^t$.
- The point $\tilde{p}_2$ is given by rotating the point $\tilde{p}_1$ by ϕ with respect to the frame $\vec{f}^t$.

Suppose that $\vec{a}^t = \vec{f}^t A$, $\tilde{p}_1 = \vec{f}^t Mc$ and $\tilde{p}_2 = \vec{f}^t Nc$.
Express the matrices *M* and *N* using the following symbols $\vec{f}^t, d, A, \theta, \phi$.

2. (30pts) Suppose a light path in the following description. (a) Compute the unknown angles θ_1, and θ_2 **(Snell's law)**. Then, (b) using Recursive Ray Tracing algorithm, determine the color at pixel P on the image plane. For reflection, use Ward model. The equation of Ward model and the coefficients of each object is listed in the table.



Name	$\frac{\rho_d}{\pi}$	ρ_s	α	etc
Object A	-	-	-	A transparent object with transmittance = 0.85
Object B	0.4	0.8	0.5	
Object C	0.85	0	0	
Light	-	-	-	Intensity I=20

Ward Model

$$f = f_d + f_s$$

$$f_d = \frac{\rho_d}{\pi},\ f_s = \frac{\rho_s}{4\pi\alpha^2\sqrt{\cos\theta_i \cos\theta_o}} e^{-\frac{\tan^2\theta_h}{\alpha^2}}$$

3. (50pts) Answer the following questions.
   a. (5 pts each) Given a random variable with the following probability density function (PDF) $p(x) = e^{-x}$ $(with\ x\ in\ [0,\infty])$.
      (1) How do you draw samples from this PDF?

      (2) Draw four samples *x1, x2, x3,* and *x4* from this distribution using the following four (uniform) random numbers *r={0.2, 0.4, 0.6, 0.8}* (with *r* in [0,1])

      (3) What is the expected value *E[f(x)]* of this distribution, where $f(x) = e^{-x}$?

   b. (10pts) How does the expected value computation relate to Monte-Carlo integration with importance sampling?

   c. (15pts) Given the integral $I = \int_0^1 4(1-x)dx$, and the PDF $p(x) = 2(1-x)$ $(x\ in\ [0,1])$. Show that this PDF is ideal for Monte Carlo integration of *I* using importance sampling.

   d. (10pts) Explain stratified sampling. Why is it better than uniform sampling?