Ph.D. Qualifying Exam — Algorithms (CS500)
Summer 2014

You must answer in *English*. Write clearly and readably!

Problem 1. (20 points) You are given a problem X. You wish to prove that X is NP-hard. Which of the following reductions can you use to prove this?

(A) X $\leq_P$ SAT   (B) 2-SAT $\leq_P$ X   (C) ShortestPath $\leq_P$ X

(D) X $\leq_P$ VertexCover   (E) 3-SAT $\leq_P$ X   (F) X $\leq_P$ ShortestPath

Problem 2. (20 points) In the problem COLORING you are given a graph G, and you want to find the smallest possible number k of colors such that the vertices of G can be colored with k colors such that no two incident vertices have the same color.

You have designed an algorithm $\mathcal{A}$ for COLORING that handles a certain class of graphs, and you want to prove that your algorithm is a 1.5-approximation. What do you have to prove?

Problem 3. (30 points) The figure below shows a flow network, with capacities on each edge.
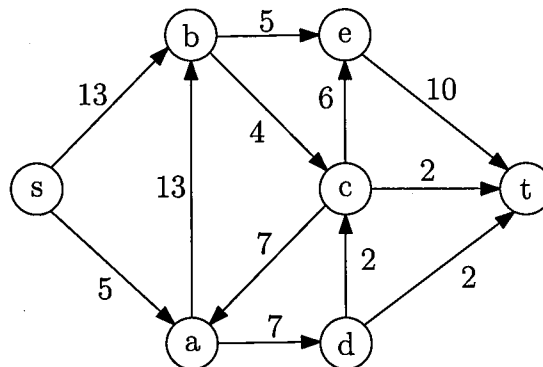
What is the *value* of the maximum flow? Give a *minimum cut* for this network.

Problem 4. (30 points) A transport plane has three cargo compartments, one in the front, one in the middle, and one at the rear end of the plane. These compartments have the following capacity in terms of weight and volume:

Compartment	Weight capacity (tons)	Volume capacity (m^3)
Front	10	6800
Center	16	8700
Rear	8	5300

There are four kinds of merchandise that you can transport:

Merchandise	Available quantity (tons)	Volume (m^3/ton)	Profit ($/ton)
M1	18	480	310
M2	15	650	380
M3	23	580	350
M4	12	390	285

The plane has to remain balanced, and so you have to load it such that each compartment carries the same percentage of its total weight capacity. For instance, if you have 5 tons in the front compartment, you must have 8 tons in the center and 4 tons in the rear compartment.

Write a linear program to select the merchandise that will maximize your profit under the given constraints. Explain clearly the meaning of each variable and each constraint. (You can assume that the merchandise can be partitioned in any way you want, for instance 5.123467 tons, or 12/7 tons, etc.)