Qualifying Exam for Algorithm. Jan., 2014.
Total Score is 120 pts. You get a pass if your get at least 60 pts.

1. (5 pts x 6 = 30 pts) Given 2 decision problems L_1 and L_2 in NP and $L_1 \leq_p L_2$ (i.e., L_1 is polynomial-time reducible to L_2), for each of the following statements, give one of T(true), F(false), or O(open question), and briefly justify your answer.
   (1) If $L_1 \in$ P, then $L_2 \in$ P.
   (2) If $L_2 \in$ P, then $L_1 \in$ P.
   (3) If $L_1 \in$ NPC, then $L_2 \in$ NPC.
   (4) If $L_2 \in$ NPC, then $L_1 \in$ NPC.
   (5) If $L_2 \leq_p L_1$, then L_1 and L_2 are NP-complete.

2. (10 pts) What is NP?

3. (10 pts) What is NP-complete?

4. (10 pts) Explain Dijkstra's algorithm for the single-source shortest path.

5. (10 pts) Explain Floyd-Warshall's all-pairs shortest path algorithm.

6. (10 pts) Explain (1) Prim's minimum spanning tree algorithm or (2) Kruskal's minimum spanning tree algorithm. Which one would you choose to implement? Why?

7. (10 pts) Describe your favorite sorting algorithm and analyze its running time. Why is it your favorite algorithm?

8. (30 pts) What is the vertex cover problem? Show that the decision version of the vertex cover problem is NP-complete. Give an approximation algorithm for the vertex cover problem.