1. (5 pts x 6 = 30 pts) Given 2 decision problems $L_1$ and $L_2$ in NP and $L_1 \leq_p L_2$ (i.e., $L_1$ is polynomial-time reducible to $L_2$), for each of the following statements, give one of T(true), F(false), or O(open question), and briefly justify your answer.
   (1) If $L_1 \in P$, then $L_2 \in P$.
   (2) If $L_2 \in P$, then $L_1 \in P$.
   (3) If $L_1 \in NPC$, then $L_2 \in NPC$.
   (4) If $L_2 \in NPC$, then $L_1 \in NPC$.
   (5) If $L_2 \leq_p L_1$, then $L_1$ and $L_2$ are NP-complete.

2. (10 pts) What is NP?

3. (10 pts) What is NP-complete?

4. (10 pts) Explain Dijkstra’s algorithm for the single-source shortest path.

5. (10 pts) Explain Floyd-Warshall’s all-pairs shortest path algorithm.

6. (10 pts) Explain (1) Prim’s minimum spanning tree algorithm or (2) Kruskal’s minimum spanning tree algorithm. Which one would you choose to implement? Why?

7. (10 pts) Describe your favorite sorting algorithm and analyze its running time. Why is it your favorite algorithm?

8. (30 pts) What is the vertex cover problem? Show that the decision version of the vertex cover problem is NP-complete. Give an approximation algorithm for the vertex cover problem.